

Tribhuvan University
Institute of Science and Technology
Model Question Paper

Bachelor Level/ First Year/ Second Semester/ Science
Computer Science and Information Technology (MTH 155)
(Linear Algebra)

Full Marks: 80

Pass Marks: 32

Time: 3 hours.

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt all questions:

Section A: Short Answer Questions.

(10x2=20)

1. If one row in an echelon form of an augmented matrix is $[0 \ 0 \ 0 \ 5 \ 0]$, then the associated linear system is inconsistent. State whether the statement is true or false. Give reasons.
2. Let $\vec{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, show that $\begin{bmatrix} h \\ k \end{bmatrix}$ is in the span $\{u, v\}$ for all h and k .
3. Let $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -5 \\ 3 & k \end{pmatrix}$, what value(s) of k , if any, will make $AB = BA$?
4. If $A = \begin{bmatrix} 6 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, then show that $\det(AB) = \det(A) \cdot \det(B)$.
5. Using Cramer's rule solve the following simultaneous equations:
$$\begin{aligned} 5x + 7y &= 3 \\ 2x + 4y &= 1 \end{aligned}$$
6. Find a matrix A such that $W = \text{Col}A$.
$$W = \left\{ \begin{bmatrix} 6a - b \\ a + b \\ -7a \end{bmatrix} : a, b \text{ in } R \right\}$$
7. The set $B = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$ is a basis for P_2 . Find the coordinate vector of $\vec{p}(t) = 1 + 4t + 7t^2$ relative to B .
8. Show that 7 is an eigen value of matrix $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$.
9. Let $\vec{v} = (1, -2, 2, 0)$. Find a unit vector $\vec{u}$ in the same direction as $\vec{v}$.
10. Let $\vec{v} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of $\vec{v}$ onto $\vec{u}$.

Section B: Brief Answer Questions.

Attempt all questions.

(5x4=20)

11. Find an equation involving g, h and k that makes the augmented matrix correspond to the consistent system.

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$$

12. Find the 3x3 matrix and corresponds to the composite transformation of a scaling by 0.3, a rotation of 90° , and finally a translation that adds $(9, -5, 2)$ to each point of a figure.
13. Find the inverse of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$, if it exists.

OR

Find a least square solution of $Ax = b$ for

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 2 \\ 5 \\ 1 \end{bmatrix}$$

14. Define null space of an $m \times n$ matrix A . prove that null space of an $m \times n$ matrix A is subspace of R^n .

15. Define Characteristics equation. Find the Characteristics equation of

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & -8 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Section C: LONG ANSWER QUESTIONS.

Attempt all questions.

(5 x 8 = 40)

16. Define linearly independent and linearly dependent set in R^n . Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$,

$$v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

- Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent.
- If possible find a linearly dependent relating among v_1, v_2, v_3 .

OR

Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$, $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ and a transformation $T : R^2 \rightarrow R^3$ defined

by $T(x) = Ax$,

- Find $T(u)$.
- Find an x in R^2 whose image under T is b .
- Is there more than one x whose image under T is b .
- Determine if c is in the range of the transformation T .

17. Define block upper triangular. Assume that A_{11} is $p \times p$, A_{22} is $q \times q$, and A is invertible. Find a formula for A^{-1} .

18. Define basis and dimension of vector space. Find the basis and dimension of the

$$\text{subspace } H = \left\{ \begin{bmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \in \mathcal{R} \right\}$$

19. Diagonalize the matrix

$$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 1 & 4 & -3 & 0 \\ -1 & -2 & 0 & -3 \end{bmatrix}$$

Find an invertible matrix P and diagonal matrix D, such that $A = PDP^{-1}$.

20. What do you understand by Gram-Schmidt process? Let $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

Then $\{x_1, x_2, x_3\}$. Using Gram-Schmidt process construct an orthogonal basis for W.

OR

Let u and v are non-zero vectors in $\mathcal{R}^2$ and $\mathcal{R}^3$ and α be the angle between u and v then prove that $u \cdot v = \|u\| \|v\| \cos \alpha$. Represent it geometrically.